



## Lobsters and Heronian Mean used for Alternative Decision making support system for Enterprising Resource Planning

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**ABSTRACT:** The Heronian mean (HM) of two non-negative real values is subject to a generalization in order to make it applicable to any subset  $a$  of  $n$  elements in the set of all positive real numbers. The generalized Heronian mean is given by  $H_\omega(a, b) = [a + b + \omega\sqrt{(ab)}] / (\omega + 2)$ . In the generalized Heronian mean which contains omega, consider omega to be an adjustment value in order to obtain the results of graceful labeling. Graceful labeling obtained by labeling the vertices with unique labels in such a way that  $\phi^*(e = uv) = [\phi(u) + \phi(v) + \omega\sqrt{\phi(uv)}] / (\omega + 2)$ , for all  $u, v$  belongs to  $V(G)$  and  $uv$  belongs to  $E(G)$ . This results in two different parameters generalized Heronian mean and graceful labeling combined as a single parameter. In this paper, proved that a family of lobster trees admit graceful labeling by the generalized Heronian mean. The application of HM is literally meant for group decision making (GDM) with multi criterion attributes for out ranking results. Using HM, it is possible to get a cognitive knowledge for labeling in a very pious manner. The safety is the first and foremost criteria for any application. In the decision making system many alternatives were available to set the policies for Enterprising Recourse Planning (ERP). The decision support system will selection a good alternative for ERP. The HM and lobster methodology labels provides the optimum path to select the good alternative path among all alternatives. For a company this method of ERP decision making help to utilize the resources in effective way which yields more cost effectiveness. By expressing unique way to find the alternatives in decision making, the lobster tree branches are different alternatives. The enterprising resource planning is obtained by using graceful labeling of lobster tree by Heronian mean in decision making.

**Keywords:** Generalized Heronian mean, graceful labeling, lobster tree, decision making, Enterprising Recourse Planning

**AMS Subject Classification:** 05C78, 26E60, 26D20

### I. INTRODUCTION

The ERP plays a vital role in all the business industries and keep the pace in setting up the policies and strategies for the industries. The plenty numbers of alternatives were available right across the industries for selecting the correct alternative. For getting right alternative to the complex problem, heronian mean will be used to find the correct solution. The decision support system was implemented with a mathematical ideology like heronian mean and lobster for labeling in 360 degrees to find the correct path among the alternatives. In the paper, the formulation of work is to select the alternative that should be the optimum path for reaching the destination in a graceful manner for the ERP decision making system.

The Heronian mean of two non-negative real values is subject to a generalization in order to make it applicable to any subset  $a$  of  $n$  elements in the set of all positive real numbers. Kaizhong Guan and Huantao [3], Zhu Walther Janous [2] defined the generalized Heronian mean as  $H_\omega(a, b) = [a + b + \omega\sqrt{(ab)}] / (\omega + 2)$ ,  $0 \leq \omega < \infty$ , where  $a, b$  are positive numbers. Walther Janous [2] discuss A Note On Generalized Heronian Means. Kaizhong Guan and Huantao Zhu [3] explained the Generalized Heronian Mean and its Inequalities.

Considering  $\omega$  as a adjusted value,  $\omega = (d - 2u) / (\sqrt{(uv)} - d)$ ,  $d = |v - u|$ ,  $u < v$  in the above formula results generalized Heronian mean for graceful labeling of graphs. In this paper, the new parameter is introduced and discuss with a family of lobster graphs.

### II. RELATED WORKS

In this paper, author emphasized about the Heronian mean and its different application is like Heronian Mean labeling, Triangular Ladder, Triangular snake, Double Triangular snake etc., [5].

In graph theory Heronian mean plays a vital role in path finding application. As such it is  $q$  complete graph that can provide a optimum solution in the congested path [6]. The paper states that the Heronian mean is applied in the segment of area unbalanced data using linguistics multiple attributes. The linguistic values always a fuzzy in nature and finding the appropriate critical path were found through the above said methodology [7]. In the multi-dimensional criterion decision making system for neutrosophic uncertainty can be identified through the Heronian mean and lobsters [8]. In coal mine application frank Heronian mean is applied to find a linguistic and intuitionistic for group decision making. Multi dimensional, multi criterion and multi objectives were incorporated for the above application [9]. In group

decision making geometric Heronian mean playing a in evitable job for out ranking the selection in hesitant fuzzy using linguistic and intuitionistic values [10]. In steganography, hiding the images over the images is the latest application done by the researchers using Heronian mean and scrambling methodology [11]. In the recent work of energy efficiency geometric Heronian mean and lobsters labeling option is the appropriate methodology deployed in the work [12]. In smart city application multiple cognitive knowledge in the segment of artificial intelligence can be implemented through frank Heronian mean for optimizing the congested atmosphere in the work [13].

### III. PROPOSED METHODOLOGIES

In this paper, the graphs are simple, finite and undirected. It consists of vertex set  $V(G)$  and edge set  $E(G)$  such that  $|V(G)| = p$  and  $|E(G)| = q$ .

The popular applications using HM is followed as such, i) Group Decision making ii) Pious labeling for identifying the person in the social media iii) Security using cryptography and iv) Reduced entropy for all types of training sets are the special features behind using HM.

#### A. Definition

Labeling of vertices in  $G$  with the numbers from 0 to  $q$  is an injective map  $\phi: V \rightarrow \{0, 1, \dots, q\}$ . It is said to be graceful labeling by the generalized Heronian mean, if there exists a labeling of its vertices such that the map  $\phi: E \rightarrow \{1, 2, \dots, q\}$  given by  $\phi^*(uv) = \frac{\phi(u) + \phi(v) + \omega\sqrt{\phi(uv)}}{\omega + 2}$ ,  $\omega$  taken as an adjustment

value  $(\omega = (d - 2u) / (\sqrt{uv} - d), d = |v - u|)$  where  $u, v \in V(G)$  and  $uv \in E(G)$  is a bijection.

#### B. Definition

A tree  $T$  is said to be a lobster tree if after deleting all of its pendant edges, it is a caterpillar tree (the removal of its end vertices leaves a path).

Heronian mean labeling graph introduced by Sandhya *et al.* discussed its limitations [5-6] and Durai [4]. The other labeling concepts were given by Durai Baskar And Manivanan, given  $F$ -Heronian Mean Labeling Of Graphs Obtained From Paths, Sandhya, *et al.*, discussed Some New Families Of Heronian Mean Graphs and Heronian mean labeling of graphs.

In Gallian's survey [1] labeling of graphs are discussed.

#### C. Main Result

**Theorem.** A family of lobster trees satisfies graceful labeling by generalized Heronian mean.

Proof:

Consider " $m_1$  branch" caterpillar. Each vertex in Level 1 merged with the center vertex of a star graph with  $m_2$  branches. This results in a Level 2 balanced lobster. It results in a lobster tree  $LT(n, 2)$ . Level 1 contains either branch or not, and level 2 have different possibilities. 1. The occurrence of branch  $T_i(2)$ , repeated an even number of times or all are isomorphic. In this paper, it is considered only that of two-level full trees  $T_1(2), T_2(2), \dots, T_n(2)$  with the above possibilities.

The following Figure shows the general form of a lobster tree.

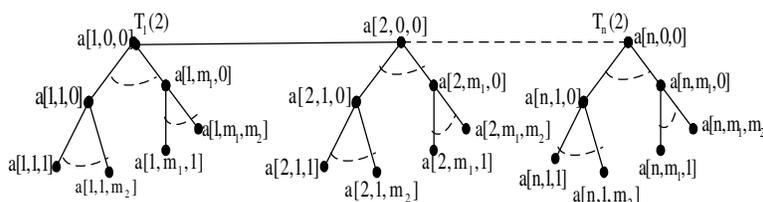


Fig. 1. General form a level 2 lobster tree.

The  $T_i(2)$ , consists of base point  $p_i$  of  $P_n$ , where  $a[i, i_1, i_2]$  are 1<sup>st</sup> level branch( $i_1$ ), 2<sup>nd</sup> level sub-branch( $i_2$ ). Let  $a[i, r, 0] = q(i, r, 0)$  denotes the number of vertices adjacent to the vertex  $r$  in level 1. Clearly  $q(i, r, 0) = q_{ir} = 1 + m(1, i) m(2, i)$ .

Hence  $E(T_i(2)) = \sum_{r=1}^{m_1} q_{ir}$ ,  $i = 1, 2, \dots, n$ . This results  $q =$

$$\sum_{i=1}^n |E(T_i(2))| + n - 1.$$

The labeling of vertices is followed.

Labeling of the 1<sup>st</sup> root branch is given under.

[1] :  $\phi(a[1, 0, 0]) = 0, \phi(a[1, 1, 0]) = q$ .

[2] :  $\phi(a[1, 1, 1]) = 1$ . Labeling of pendent vertices are increased by 1 up to  $m_2$ .

Labeling of 1<sup>st</sup> intermediary branches are as follows:-

[3] :  $\phi(a[1, r, 0]) = \phi(a[1, (r-1), 0]) - q_{1(r-1)}, 2 \leq r \leq m_1$ .

The leftmost free leaf in each adjacent branches are as follows :-

[4] :  $\phi(a[1, j, 1]) = \phi(a[1, (j-1), 1]) + q_{1(r-1)}, 2 \leq j \leq m_1$ .

Labeling of pendent vertices is increased by 1 up to  $m_2$ .

Now, labeling of the 2<sup>nd</sup> root branch is as follows:-

[1] :  $\phi(a[2, 0, 0]) = q - |E(T_1(s))|, \phi(a[2, 1, 0, 0]) = |E(T_1(s))|$ .

[2] :  $\phi(a[2, 1, 1]) = \phi(a[2, 0, 0]) + 1$ . Labeling of pendent vertices are increased by 1 up to  $m_2$ .

Labeling of 2<sup>nd</sup> intermediary branches are as follows:-

[3] :  $\phi(a[2, r, 0]) = \phi(a[2, (r-1), 0]) - q_{2(r-1)}, 2 \leq r \leq m_2$ .

The leftmost free leaf in each adjacent branches are as follows :-

[4] :  $\phi(a[2, j, 1]) = \phi(a[2, (j-1), 1]) + q_{2(r-1)}, 2 \leq j \leq m_2$ .

Labeling of pendent vertices is increased by 1 up to  $m_2$ .

(Let  $n = 2\lambda + 1$  for odd and  $n = 2\lambda$  for even)

Case 1:  $n$  is odd, then

Odd segments of  $T(n, 2)$ :

Labeling of odd part of the root branch is as follows:-

$$[1] : \phi(a[2d + 1, 0, 0]) = \phi(a[2d - 1, 0, 0]) + (|E(T_{2d}(s))|) + 1, 1 \leq d \leq \lambda$$

$$[2] : \phi(a[2d + 1, 1, 0]) = q - \phi(a[2d + 1, 0, 0]), 1 \leq d \leq \lambda$$

$$[3] : \phi(a[2d + 1, 1, 1]) = \phi(a[2d - 1, 0, 0]) + 1, 1 \leq d \leq \lambda.$$

Labeling of pendent vertices are increased by 1 up to  $m_{2d+1}$ .

Labeling of odd part of intermediary branches are as follows:-

$$\phi(a[2d + 1, r, 0]) = \phi(a[2d + 1, (r - 1), 0]) - q_{2d+1(r-1)}, 2 \leq r \leq m_{2d+1}, 1 \leq d \leq \lambda$$

The leftmost free leaf in each adjacent branches odd part are as follows :-

$$\phi(a[2d + 1, j, 1]) = \phi(a[2d + 1, j - 1, 1]) + q_{2d+1(r-1)}, 2 \leq j \leq m_{2d+1}.$$

Labeling of pendent vertices is increased by 1 up to  $m_{2d+1}$ .

Even segments of  $T(n, 2)$  are given below.

Labeling of even part of the root branch is as follows:-

$$[1] : \phi(a[2d + 2, 0, 0]) = \phi(a[2d, 0, 0]) - |E(T_{2d+1}(s))| - 1, 1 \leq d \leq \lambda - 1.$$

$$[2] : \phi(a[2d + 2, 1, 0]) = q - \phi(a[2d + 2, 0, 0]), 1 \leq d \leq \lambda - 1.$$

$$[3] : \phi(a[2d + 2, 1, 1]) = \phi(a[2d + 2, 0, 0]) + 1, 1 \leq d \leq \lambda - 1.$$

Labeling of pendent vertices are increased by 1 up to  $m_{2d+2}$ .

Labeling of even part of intermediary branches are as follows:-

$$\phi(a[2d + 2, r, 0]) = \phi(a[2d + 2, (r - 1), 0]) - q_{2d+2(r-1)}, 2 \leq r \leq m_{2d+2}.$$

The leftmost free leaf in each adjacent branches of even part are as follows :-

$$\phi(a[2d + 2, j, 1]) = \phi(a[2d + 2, j - 1, 1]) + q_{2d+2(r-1)}, 2 \leq j \leq m_{2d+2}.$$

Labeling of pendent vertices is increased by 1 up to  $m_{2d+2}$ .

Case 2:  $n$  is even, then

Odd segments of  $T(n, 2)$ :

Labeling of odd part of the root branch is as follows:-

$$[1] : \phi(a[2d + 1, 0, 0]) = \phi(a[2d - 1, 0, 0]) + (|E(T_{2d}(s))|) + 1, 1 \leq d \leq \lambda - 1.$$

$$[2] : \phi(a[2d + 1, 1, 0]) = q - \phi(a[2d + 1, 0, 0]), 1 \leq d \leq \lambda - 1.$$

$$[3] : \phi(a[2d + 1, 1, 1]) = \phi(a[2d - 1, 0, 0]) + 1, 1 \leq d \leq \lambda - 1.$$

Labeling of pendent vertices are decreased by 1 up to  $m_{2d+1}$ .

$$[4] : \phi(a[2d + 1, j, 1]) = \phi(a[2d + 1, j - 1, 1]) - q_{2d+1(r-1)}, 1 \leq d \leq \lambda - 1, 2 \leq j \leq m_{2d+1}.$$

Labeling of pendent vertices are decreased by 1 up to  $m_{2d+1}$ .

Labeling of odd part of intermediary branches are as follows:-

$$\phi(a[2d + 1, r, 0]) = \phi(a[2d + 1, (r - 1), 0]) - q_{2d+1(r-1)}, 2 \leq r \leq m_{2d+1}.$$

Even segments of  $T(n, 2)$ :

Labeling of even part of root branches are as follows:-

$$[1] : \phi(a[2d + 2, 0, 0]) = \phi(a[2d, 0, 0]) - (|E(T_{2d}(s))|) - 1, 1 \leq d \leq \lambda - 1.$$

$$[2] : \phi(a[2d + 2, 1, 0]) = q - \phi(a[2d + 2, 0, 0]), 1 \leq d \leq \lambda - 1.$$

$[3] : \phi(a[2d + 2, 1, 1]) = \phi(a[2d + 2, 0, 0]) + 1, 1 \leq d \leq \lambda - 1.$  Labeling of pendent vertices are increased by 1 up to  $m_{2d+2}$ .

The leftmost free leaf in each adjacent branches from the Level 2 are as follows :-

$$\phi(a[2d + 2, j, 1]) = \phi(a[2d + 2, j - 1, 1]) + q_{2d+2(r-1)}, 1 \leq d \leq \lambda - 1, 2 \leq j \leq m_{2d+2}.$$

Labeling of pendent vertices are increased by 1 up to  $m_{2d+2}$ .

Labeling of even part of intermediary branches are as follows:-

$$\phi(a[2d + 2, r, 0]) = \phi(a[2d + 2, (r - 1), 0]) + q_{2d+2(r-1)}, 2 \leq r \leq m_{2d+2}.$$

To prove  $\phi$  is a bijection, assume two vertices  $e_1 = v_{ii}$  and  $e_2 = v_{jm}$  respectively.

case 1: consider  $i, j$  be even. let  $i = 2k + 2, j = 2k' + 2, l = 2k + 2$  and  $m = 2k' + 2$ .

$$\text{let } k' > k. \text{ Then } \phi(v_{ii}) - \phi(v_{jm}) = q - 3 \sum_{i=1}^{2k+1} q_i - 3(k + 1) -$$

$$2q_{2k+1} - l_{j-1} - [q - 3 \sum_{i=1}^{2k'+1} q_i - 3(k' + 1) - 2q_{2k'+1} - l'_j - 1]$$

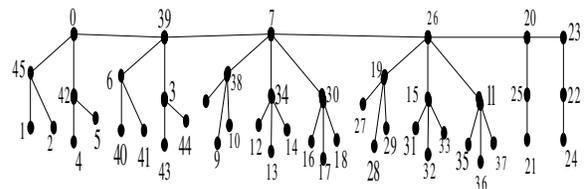
$$= 3 \sum_{i=2k+1}^{2k'+2} q_i - 3(k' - k) + l_j - j > 0.$$

This results  $e_1 \neq e_2$  if and only if  $\phi(e_1) \neq \phi(e_2)$ . Similarly, other cases can be proved. Therefore  $\phi$  is a bijection. It gives all the vertices are labeled with different values and resultant edge values are different results uniqueness of graceful labeling.

#### IV. RESULTS AND DISCUSSIONS

Graceful labeling gives uniqueness in the evaluation of graph structure, this yields a new way to solve real-time application of ERP for selecting the correct alternative among the huge alternatives in the areas like linguistic multiple attribute decision-making, applications in decision making systems.

This can be viewed in the following diagram with sum of two adjacent vertices label value equal to  $q$ , is nothing but the optimum path with least to higher alternatives.



## V. CONCLUSION

In the paper, a new parameter is introduced by combining two different areas like, General form of level 2 lobster trees and graceful lobster tree discussed in the outcome. A family of lobster trees admits graceful labeling by the generalized Heronian mean for the above said applications. It paves the way to a new area of research in pure and application areas. The correlation and regression factors are taken into account for measuring the security and safety behind graceful labeling. Most of the organizations believe only on mathematical modelling that gives mileage for selection of good alternative in decision making problems via graceful labeling. The researchers finding a new path in decision making through the mathematical modelling HM. In HM, complex formulation of mathematical ideology will provide a greater strung in cryptography and network security. In a minimizing the vertex and edges, it is possible to map all criteria's for taking a good decision making using linguistic and intuitionist.

## CONFLICT OF INTEREST

The authors declares that there is no conflict of interest.

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